Resolución de Sistemas de Ecuaciones Diferenciales por el método de Matriz Exponencial

SISTEMA. $\dot{\bar{x}} = A\bar{x} + \bar{b}(t)$ S(n) EDO(1) LCC NH.

solución. $\bar{x} = e^{At}\bar{x}(0) + e^{At}\int_0^t e^{-A\tau}\,\bar{b}(\tau)\,d\tau$

Propiedades de la Matriz Exponencial e^{At}

1) $t=0$ $\quad e^{A(0)} \Rightarrow e^{(0)} \Rightarrow I$

2) $$\frac{de^{At}}{dt} = Ae^{At}$$

3) $$e^{(A+B)t} = e^{At} \cdot e^{Bt}$$

4) $$e^{At} \cdot e^{-At} = I$$

$$e^{At} = I + At + \frac{A^2}{2!}t^2 + \frac{A^3}{3!}t^3 + \dots + \frac{A^n}{n!}t^n + \dots$$

$$e^{At} = B_0 I + B_1 A + B_2 A^2 + \dots + B_{n-1} A^{n-1}$$

ecuación característica $\quad \det(A - \lambda I) = 0 \quad \lambda_i$

$$e^{\lambda_i t} = 1 + \lambda_i t + \frac{\lambda_i^2}{2!}t^2 + \frac{\lambda_i^3}{3!}t^3 + \dots + \frac{\lambda_i^n}{n!}t^n + \dots$$

$$e^{\lambda_1 t} = B_0 + B_1 \lambda_1 + B_2 \lambda_1^2 + \dots + B_{n-1} \lambda_1^{n-1}$$

$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ obtener la Matriz Exponencial

$$e^{At} = B_0 I + B_1 A$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(3-\lambda)-(2)(4)=0$$

$$\lambda^2-4\lambda+3-8=0$$

$$\lambda^2-4\lambda-5=0$$

$$(\lambda-5)(\lambda+1)=0$$

$$\boxed{\begin{array}{l} \lambda_1=-1 \\ \lambda_2=5 \end{array}}$$

$$\left.\begin{aligned} e^{5t} &= B_0 + 5B_1 \\ e^{-t} &= B_0 - B_1 \end{aligned}\right\} \qquad \begin{aligned} B_0 &= \frac{1}{6}\left(e^{5t} + 5e^{-t}\right) \\ B_1 &= \frac{1}{6}\left(e^{5t} - e^{-t}\right) \end{aligned}$$

$$e^{At} = \frac{1}{6}\left(e^{5t} + 5e^{-t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{6}\left(e^{5t} - e^{-t}\right)\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$e^{At} = \frac{1}{6}\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} e^{5t} + \frac{1}{6}\begin{bmatrix} 4 & -2 \\ -4 & 2 \end{bmatrix} e^{-t}$$